HIRDLS

SW-HIR-2030

HIGH RESOLUTION DYNAMICS LIMB SOUNDER

Originator: Tracy Pedersen Date: 04/27/2004

Subject/Title: **HIRDLS Pointing Accuracy & Knowledge Analysis**

Document is composed of several memos that were generated by NASA (Tracy Pedersen).

Keywords:

Purpose of this Document:

Reviewed by: Date (yy-mm-dd):			

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EOS

4-27-04

Glenn and Claire,

Attached you will find the final release of the HIRDLS Pointing Accuracy and Knowledge Analysis. As you will recall, these documents provide an evaluation of whether the HIRDLS instrument satisfies various alignment requirements given in the ITS, UIID, and ICD. Final release of the analysis was delayed pending determination of the azimuth sign convention used by SSG during calibration of the optical bench. (The sign convention was determined just a few weeks ago, prior to the AURA Observatory Pre-Ship Review). The attached documents are:

Technical Memorandum HIRDLS-TP-147C, Verification of HIRDLS ITS Alignment Specifications
Technical Memorandum HIRDLS-TP-148C, HIRDLS Line of Sight Coordinate Transformation
Technical Memorandum HIRDLS-TP-149C, Verification of HIRDLS UIID Alignment Specification
Technical Memorandum HIRDLS-TP-150A, Summary of HIRDLS Pointing Accuracy and Knowledge

Following is a summary of specifications evaluated by the alignment analysis and results of the analysis:

ITS 3.5.1.1 Elevation Scan Range	
Boresight to IRCF	waiver necessary
ILOS Scan Range relative to Boresight	specification met (Note 1)
ITS 3.5.2.1 Azimuth Scan Range	
Boresight to IRCF	specification met
ILOS Scan Range relative to Boresight	specification met (Note 1)
ITS 3.5.3.2 TRCF to IRCF Alignment	
X-axis	waiver necessary
Y-axis	waiver necessary
Z-axis	specification met
UIID 3.1.4.3 (Pointing Knowledge) Instrument Allocation (CCR 424-12-21-128, W-123) (Note 2)	
X-axis	waiver necessary
Y-axis	waiver necessary
Z-axis	specification met

As can be seen above, Change Requests (Waivers) are necessary for three specifications. These unmet specifications have been identified in previous communications. As of today, the Change Request for UIID 3.1.4.3 is in final review, and waivers for ITS 3.5.1.1 and ITS 3.5.3.2 have not yet been submitted by Lockheed Martin.

The results of the HIRDLS pointing analysis have already been incorporated in Lockheed Martin document TR-LOC-1276. Therefore, other than the two remaining Change Requests for the specifications listed above, I believe that no additional actions are required due to this final release of the HIRDLS Pointing Accuracy and Knowledge Analysis. The results of the analysis will, of course, find further use in post-launch data processing.

Thanks,
Tracy

Note 1: These specifications were verified elsewhere than the HIRDLS Pointing Accuracy and Knowledge Analysis.

Note 2: ICD paragraph 3.5.3.2 Accuracy, Knowledge, and Stability Allocation contains the same requirements (among others) as UIID paragraph 3.1.4.3 (Pointing Knowledge) Instrument Allocation.

Technical Memorandum

From: Tracy R. Pedersen, Swales Aerospace

To: Glenn B.Jackson, Claire L. Wilda

Date: April 27, 2004

Subject: **Verification of HIRDLS ITS Alignment Specifications**

Document No: HIRDLS-TP-147C

Requirements:

HIRDLS alignment requirements, taken from the HIRDLS Instrument Technical Specification (ITS, Document # GSFC 424-28-21-13) are given in matrices Req1 and Req2dms. The values given in matrix Req1 represent the required placement of the Line of Sight (LOS) relative to the Instrument Reference Coordinate Frame (IRCF), in decimal degrees of azimuth and elevation. The values given in matrix Req1Int represent the size of the allowable azimuth and elevation angular intervals.The values given in matrix Req2dms represent the maximum allowable rotation of the TRCF relative to the IRCF in arcsec of rotation around the indicated axes. The values in matrix Req2 are the values in matrix Req2dms expressed in decimal degrees. The sense of rotation may be either positive or negative, so the interval of allowable rotation is twice that given in matrix Req2dms, and is shown in decimal degrees in matrix Req22.

$$\mathrm{Req1} := \begin{pmatrix} -0.1 & 0.1 \\ 25.217 & 25.383 \end{pmatrix}$$

min and max LOS to IRCF azimuth (ITS 3.5.2.1)
min and max LOS to IRCF elevation (ITS 3.5.1.1)

$$\mathrm{Req1Int} := \begin{pmatrix} \mathrm{Req1}_{1,2} - \mathrm{Req1}_{1,1} \\ \mathrm{Req1}_{2,2} - \mathrm{Req1}_{2,1} \end{pmatrix} \qquad \mathrm{Req1Int} = \begin{pmatrix} 0.200 \\ 0.166 \end{pmatrix}$$

$$\mathrm{Req2dms} := \begin{pmatrix} 204 \\ 240 \\ 204 \end{pmatrix}$$

max allowable rotation of the TRCF around the IRCF x axis (ITS 3.5.3.2)
max allowable rotation of the TRCF around the IRCF y axis (ITS 3.5.3.2)
max allowable rotation of the TRCF around the IRCF z axis (ITS 3.5.3.2)

$$\mathrm{Req2} := \frac{\mathrm{Req2dms}}{3600} \qquad \mathrm{Req2} = \begin{pmatrix} 0.057 \\ 0.067 \\ 0.057 \end{pmatrix} \qquad \mathrm{Req22} := 2 \cdot \mathrm{Req2} \qquad \mathrm{Req22} = \begin{pmatrix} 0.113 \\ 0.133 \\ 0.113 \end{pmatrix}$$

Placement of the LOS relative to the IRCF:

$l := 3$ $\quad i := 1, 2 .. l$ $\quad$ l is the number of parameters

$n := 4$ $\quad j := 1, 2 .. n$ $\quad$ n is the number of samples

Vlos_ircf is a matrix containing the x, y, and z components of unit vectors along the measured LOS that have been transformed into the IRCF. The data in matrix Vlos_ircf are derived in Technical Memorandum HIRDLS-TP-148C.

$\boxed{Vlos\_ircf := READPRN(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Vlos_ircf"})}$

$$Vlos\_ircf = \begin{pmatrix} -0.905196 & -0.904612 & -0.904492 & -0.904322 \\ -0.000680 & -0.000756 & -0.000625 & -0.000497 \\ 0.424993 & 0.426236 & 0.426490 & 0.426852 \end{pmatrix}$$

LOS unit vector x component in the IRCF

LOS unit vector y component in the IRCF

LOS unit vector z component in the IRCF

Converting these values to azimuth and elevation gives:

$$EL_j := \frac{180}{\pi} \cdot asin(Vlos\_ircf_{3,j}) \qquad EL = \begin{pmatrix} 25.150 \\ 25.229 \\ 25.245 \\ 25.268 \end{pmatrix}$$

$$AZ_j := \frac{180}{\pi} \cdot \left(asin\left(\frac{-Vlos\_ircf_{2,j}}{cos\left(EL_j \cdot \frac{\pi}{180}\right)} \right) \right) \qquad AZ = \begin{pmatrix} 0.043 \\ 0.048 \\ 0.040 \\ 0.031 \end{pmatrix}$$

$$AZEL := stack(AZ^T, EL^T) \qquad AZEL = \begin{pmatrix} 0.043 & 0.048 & 0.040 & 0.031 \\ 25.150 & 25.229 & 25.245 & 25.268 \end{pmatrix}$$

Solving for the probabilities that the instrument LOS azimuth and elevation will be within the specified interval:

The number of samples of each alignment measurement, four, is quite small. For such cases, even though the underlying population may be Gaussian, a Gaussian distribution cannot be used to accurately derive probability intervals. In such cases, a t-distribution is often used to model the sample population. A t-distribution will be used here.

Use of the t-distribution with sample values that are close to zero, such as the azimuth data, will result in significant errors. In order to eliminate such errors, an arbitrary value of ten will be added to the azimuth values.

$l := 2$ $\quad i := 1, 2 .. l$

$n := 2$ $\quad j := 1, 2 .. n$ $\qquad Req1_{1,j} := Req1_{1,j} + 10 \qquad Req1 = \begin{pmatrix} 9.900 & 10.100 \\ 25.217 & 25.383 \end{pmatrix}$

$l := 2$ $\quad i := 1, 2 .. l$

$n := 4$ $\quad j := 1, 2 .. n$ $\qquad AZEL_{1,j} := AZEL_{1,j} + 10 \qquad AZEL = \begin{pmatrix} 10.043 & 10.048 & 10.040 & 10.031 \\ 25.150 & 25.229 & 25.245 & 25.268 \end{pmatrix}$

The mean value, m, of the parameters are:

$$m_i := \frac{1}{n} \cdot \sum_{j=1}^{n} AZEL_{i,j} \qquad m = \begin{pmatrix} 10.040 \\ 25.223 \end{pmatrix}$$

The standard deviations, s, of the parameter are:

$$s_i := \left[\frac{1}{n-1} \cdot \sum_{j=1}^{n} \left(AZEL_{i,j} - m_i \right)^2 \right]^{0.5} \qquad s = \begin{pmatrix} 0.007 \\ 0.051 \end{pmatrix}$$

The degree of freedom, f, is:

$f := n - 1 \qquad f = 3$

The mean occurrence rates, v, are:

$$v(i) := \frac{n}{m_i} \qquad v(i) = \begin{pmatrix} 0.398 \\ 0.159 \end{pmatrix}$$

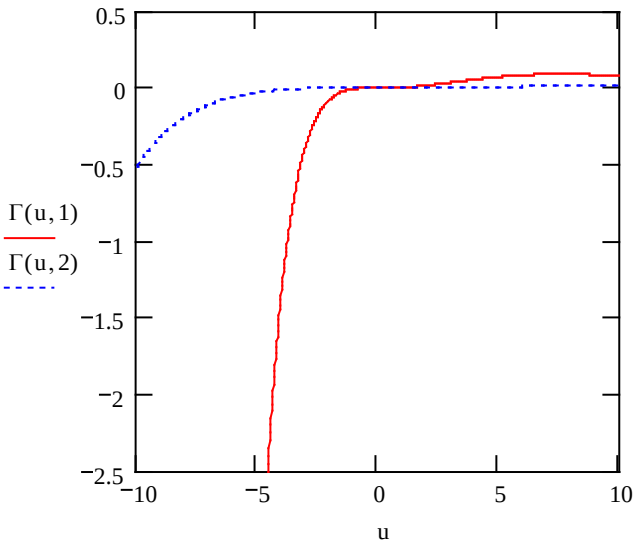
The Gamma probability density function, Γ, is:

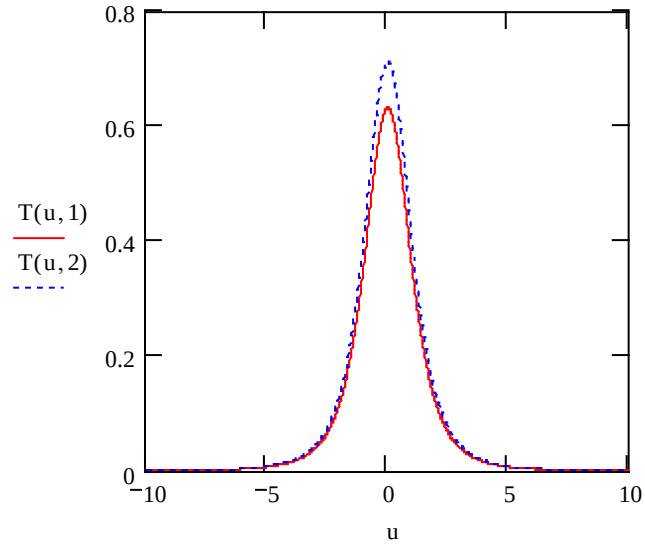
$$\Gamma(u, i) := \frac{v(i) \cdot (v(i) \cdot u)^{n-1}}{(n-1)!} \cdot e^{-v(i) \cdot u}$$

Where u is a variable representing interval.

The T probability density function, T, is:

$$T(u,i) := \frac{\Gamma\left(\frac{f+1}{2}, i\right)}{(\pi \cdot f)^{0.5} \cdot \Gamma\left(\frac{f}{2}, i\right)} \cdot \left(1 + \frac{u^2}{f}\right)^{\frac{-1}{2} \cdot (f+1)}$$





The probability, P, that the value of a parameter will be in the interval umin to umax is:

$$P(u,i) := \frac{\int_{umin_i}^{umax_i} T(u,i)\,du}{\int_{-100}^{100} T(u,i)\,du}$$

Solving for u, umin, and umax using standard formulas for interval estimation gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 0 \\ \text{while } 2 \cdot u_i \cdot \frac{s_i}{n^{0.5}} \le Req1Int_i \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u_i = \begin{pmatrix} 28.943 \\ 3.248 \end{pmatrix}$$

$$umin_i := \left[\left(Req1_{i,1} + 0\right) - m_i\right] \cdot \frac{n^{0.5}}{s_i} \qquad umin = \begin{pmatrix} -40.661 \\ -0.236 \end{pmatrix}$$

$$umax_i := \left[\left(Req1_{i,2} + 0\right) - m_i\right] \cdot \frac{n^{0.5}}{s_i} \qquad umax = \begin{pmatrix} 17.223 \\ 6.260 \end{pmatrix}$$

Solving for P gives:

$$P(u,i) := \frac{\int_{umin_i}^{umax_i} T(u,i)\,du}{\int_{-100}^{100} T(u,i)\,du} \qquad P(u,i) = \begin{pmatrix} 1.000 \\ 0.581 \end{pmatrix}$$

$l := 2 \qquad i := 1, 2 .. l$

$n := 2 \qquad j := 1, 2 .. n \qquad Req1_{1,j} := Req1_{1,j} - 10$

*
* Probability that the azimuth will fall within the specified interval of $Req1 = \begin{pmatrix} -0.100 & 0.100 \\ 25.217 & 25.383 \end{pmatrix}$ deg is $P(u,i) = \begin{pmatrix} 1 \\ 0.581 \end{pmatrix}$ *
* Probability that the elevation will fall within the specified interval of *
*

Rotation of the TRCF relative to the IRCF:

$l := 3$ $\quad i := 1, 2 .. l$ $\quad$ l is the number of parameters

$n := 4$ $\quad j := 1, 2 .. n$ $\quad$ n is the number of samples

ROTxyz is a matrix containing the rotations of the TRCF about the x, y, and z axes of the IRCF. The data in matrix ROTxyz are derived in Technical Memorandum HIRDLS-TP-148C.

$$ROTxyz := READPRN(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\ROTxyz"})$$

$$ROTxyz = \begin{pmatrix} -0.190556 & -0.208889 & -0.181944 & -0.177778 \\ 0.127778 & 0.051111 & 0.035000 & 0.011111 \\ -0.057500 & -0.054444 & -0.057778 & -0.050556 \end{pmatrix}$$

Solving for the probabilities that the rotations of the TRCF relative to the IRCF will be within the specified bounds:

The number of samples of each alignment measurement, four, is quite small. For such cases, even though the underlying population may be Gaussian, a Gaussian distribution cannot be used to accurately derive probability intervals. In such cases, a t-distribution is often used to model the sample population. A t-distribution will be used here.

Use of the t-distribution with sample values that are close to zero, such as the data in matrix ROTxyz, will result in significant errors. In order to eliminate such errors, an arbitrary value of ten will be added to all values.

$$ROTxyz := ROTxyz + 10 \qquad ROTxyz = \begin{pmatrix} 9.809 & 9.791 & 9.818 & 9.822 \\ 10.128 & 10.051 & 10.035 & 10.011 \\ 9.943 & 9.946 & 9.942 & 9.949 \end{pmatrix}$$

$l := 3$ $\quad i := 1, 2 .. l$ $\quad$ l is the number of parameters

$n := 4$ $\quad j := 1, 2 .. n$ $\quad$ n is the number of samples

The mean value, m, of the parameter is:

$$m_i := \frac{1}{n} \cdot \sum_{j=1}^{n} ROTxyz_{i,j} \qquad m = \begin{pmatrix} 9.810 \\ 10.056 \\ 9.945 \end{pmatrix}$$

The standard deviation, s, of the parameter is:

$$s_i := \left[\frac{1}{n-1} \cdot \sum_{j=1}^{n} \left(ROTxyz_{i,j} - m_i \right)^2 \right]^{0.5} \qquad s = \begin{pmatrix} 0.014 \\ 0.05 \\ 0.003 \end{pmatrix}$$

The degree of freedom, f, is:

$$f := n - 1 \qquad f = 3$$

The mean occurrence rate, v, is:

$$v(i) := \frac{n}{m_i} \qquad v(i) = \begin{pmatrix} 0.408 \\ 0.398 \\ 0.402 \end{pmatrix}$$

The Gamma probability density function, Γ, is:

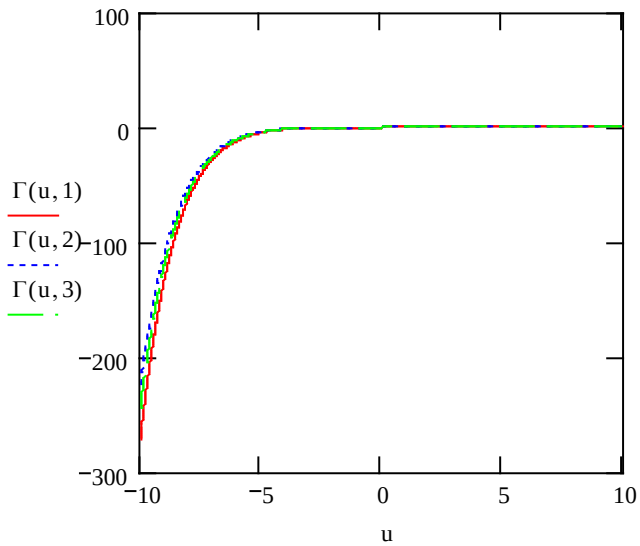
$$\Gamma(u, i) := \frac{v(i) \cdot (v(i) \cdot u)^{n-1}}{(n-1)!} \cdot e^{-v(i) \cdot u}$$

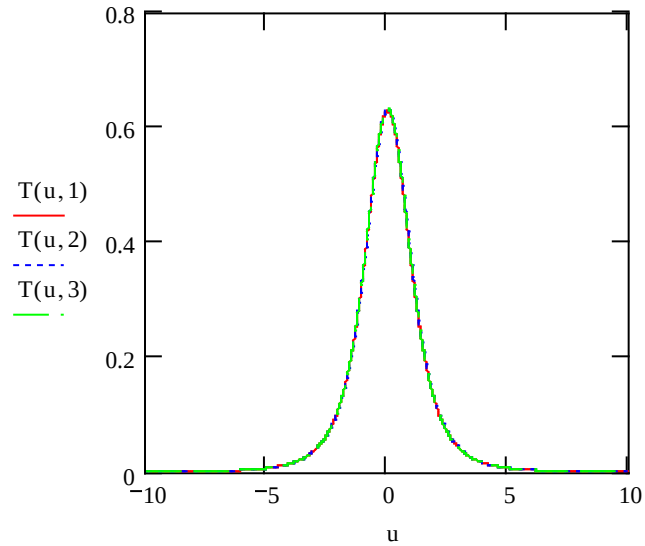
Where u is a variable representing interval.

The T probability density function, T, is:

$$T(u, i) := \frac{\Gamma\left(\frac{f+1}{2}, i\right)}{(\pi \cdot f)^{0.5} \cdot \Gamma\left(\frac{f}{2}, i\right)} \cdot \left(1 + \frac{u^2}{f}\right)^{\frac{-1}{2} \cdot (f+1)}$$

$u := -10, -9.99 .. 10$





The probability, P, that the value of a parameter will be in the interval umin to umax is: $P(u,i) := \dfrac{\int_{umin_i}^{umax_i} T(u,i)\,du}{\int_{-100}^{100} T(u,i)\,du}$

Solving for u, umin, and umax using standard formulas for interval estimation gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 0 \\ \text{while } 2 \cdot u_i \cdot \dfrac{s_i}{n^{0.5}} \le Req22_i \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u_i = \begin{pmatrix} 8.214 \\ 2.644 \\ 33.661 \end{pmatrix}$$

$$umin_i := -u_i \qquad umin = \begin{pmatrix} -8.214 \\ -2.644 \\ -33.661 \end{pmatrix}$$

$$umax_i := u_i \qquad umax = \begin{pmatrix} 8.214 \\ 2.644 \\ 33.661 \end{pmatrix}$$

Solving for P gives: $P(u,i) := \dfrac{\int_{umin_i}^{umax_i} T(u,i)\,du}{\int_{-100}^{100} T(u,i)\,du}$ $\qquad P(u,i) = \begin{pmatrix} 0.996 \\ 0.923 \\ 1.000 \end{pmatrix}$

* Probability that x axis rotation will fall within the specified interval of +/-
* Probability that y axis rotation will fall within the specified interval of +/-
* Probability that z axis rotation will fall within the specified interval of +/-

$Req2_i = \begin{pmatrix} 0.057 \\ 0.067 \\ 0.057 \end{pmatrix}$ deg is $P(u,i) = \begin{pmatrix} 0.996 \\ 0.923 \\ 1 \end{pmatrix}$ *

Technical Memorandum

From: Tracy R. Pedersen, Swales Aerospace

To: Glenn B. Jackson, Claire L. Wilda

Date: April 27, 2004

Subject: **HIRDLS Line of Sight Coordinate Transformation**

Document No: HIRDLS-TP-148C

In order to determine the direction of the instrument Line of Sight (LOS) relative to the Instrument Alignment Cube (IAC), the known orientation of the LOS relative to the OBAC must be transformed to an orientation relative to the IAC. This will be accomplished through the following steps:

1) The measured azimuth and elevation of the LOS relative to the OBAC will be converted to the x, y, and z components of unit vectors in the direction of the LOS and relative to the OBAC.

2) The LOS unit vector relative to the OBAC will be transformed into a unit vector relative to the IRCF.

3) The LOS unit vector relative to the IRCF will be transformed into a unit vector relative to the IAC.

The Spacecraft Reference Coordinate Frame (SRCF), the Instrument Reference Coordinate Frame (IRCF), and the Telescope Reference Coordinate Frame (TRCF) are all right handed rectangular systems. These coordinate systems are all nominally identical with the exception that the origins of all three coordinate systems are different. The HIRDLS instrument contains two optical alignment cubes called the Interface Alignment Cube (IAC) and the Optical Bench Assembly Cube (OBAC). The IAC is used to define the orientation of the IRCF when the instrument is mated to the spacecraft. Three dowel pins are used to define the orientation the IRCF when the instrument is not mated to the spacecraft. The OBAC is used to define the orientation the TRCF when the OBA is mated to the instrument. Three fiducial marks on the OBA are used to define the orientation of the TRCF when the OBA is not mated to the instrument. The IAC primary surface is nominally offset from the y-z plane of the IRCF by 30 degrees (clockwise when looking down the IRCF +z axis toward the IRCF origin). The IAC secondary surface is nominally offset from the x-z plane of the IRCF by 30 degrees (clockwise when looking down the IRCF +z axis toward the IRCF origin). The OBAC primary surface is nominally parallel to the y-z plane of the TRCF. The OBAC secondary surface is nominally parallel to the x-z plane of the TRCF.

Knowledge of the orientations of the SCRF and TRCF are not necessary to the following analysis, and are described here for reference only.

For the purpose of this analysis, two additional coordinate systems shall be defined. Those are:

1) Instrument Alignment Cube Coordinate System (IACCF): The IACCF is a right handed rectangular system. The +x axis of the IACCF is nominally perpendicular to, and pointing into, the IAC primary surface; the +y axis is nominally perpendicular to, and pointing out of, the IAC secondary surface; and the +z axis is nominally parallel to, and in the same direction as, the +z axis of the IRCF.

2) Optical Bench Alignment Cube Coordinate System (OBACCF): The OBACCF is a right handed rectangular system. The +x axis of the OBACCF is nominally perpendicular to, and pointing out of, the OBAC primary surface; the +y axis is nominally perpendicular to, and pointing into, the OBAC secondary surface; and the +z axis is nominally parallel to, and in the same direction as, the +z axis of the IRCF.

The nominal relative orientations of the IRCF, TRCF, IACCF, OBACCF, IAC, and OBAC are shown in Figure 1 (top view) and Figure 2 (side view).

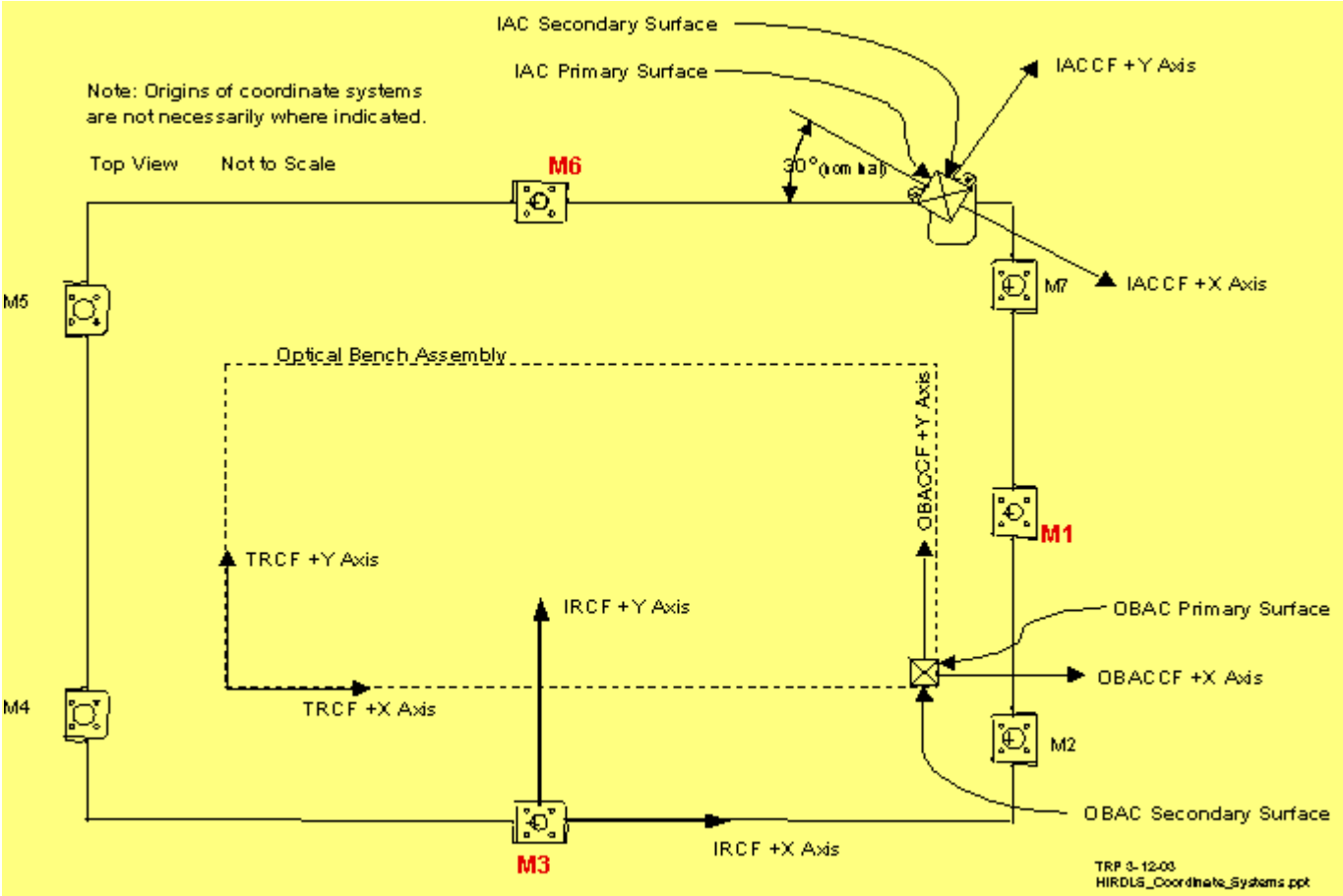


Figure 1: Top view, nominal alignment of HIRDLS coordinate system axes and reference cubes.

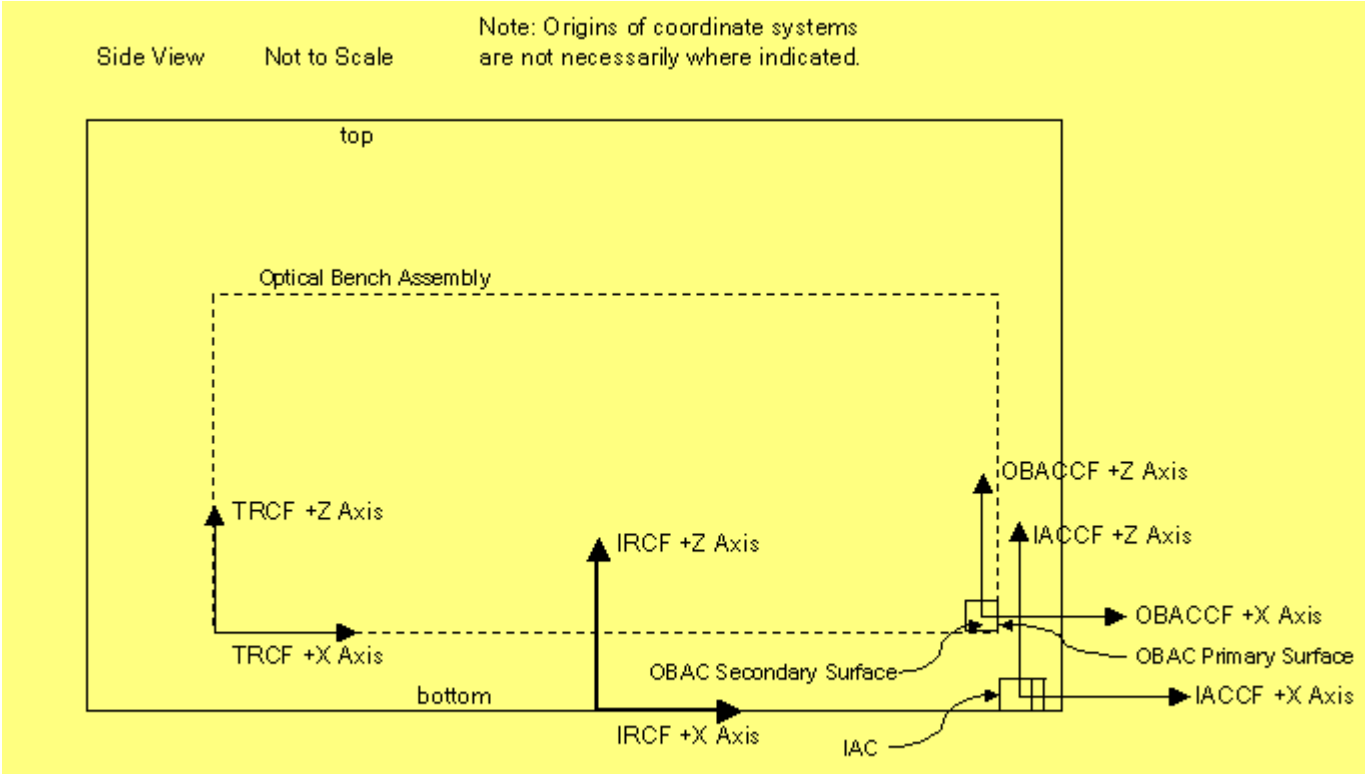


Figure 2: Side view, nominal alignment of HIRDLS coordinate system axes and alignment cubes.

Determination of the LOS unit vector components in the OBACCF:

$n := 3$ $\quad i := 1, 2 .. n$ $\quad$ n is the number of measured parameters

$m := 4$ $\quad j := 1, 2 .. m$ $\quad$ m is the number of sets of independent measurements

The matrix Assg contains the results of HIRDLS alignment measurements performed by SSG Inc. The data in matrix Assg are measurements of the LOS azimuth and elevation in decimal degrees. The orientation of the LOS was measured four times.

$$Assg := \begin{pmatrix} -0.104 & -0.105 & -0.104 & -0.103 \\ 25.278 & 25.280 & 25.280 & 25.279 \end{pmatrix}$$

Azimuth of the LOS from the OBACCF x-z plane
Elevation of the LOS from the OBACCF x-y plane

Converting the LOS azimuth and elevation to x, y, and z components in the OBACCF gives:

$$x_j := -\cos\left(Assg_{2,j} \cdot \frac{\pi}{180}\right) \cdot \cos\left(Assg_{1,j} \cdot \frac{\pi}{180}\right)$$

$$y_j := -\cos\left(Assg_{2,j} \cdot \frac{\pi}{180}\right) \cdot \sin\left(Assg_{1,j} \cdot \frac{\pi}{180}\right)$$

$$z_j := \sin\left(Assg_{2,j} \cdot \frac{\pi}{180}\right)$$

$$Vlos\_obaccf := stack\left(x^T, y^T, z^T\right)$$

$$Vlos\_obaccf = \begin{pmatrix} -0.904245 & -0.90423 & -0.90423 & -0.904238 \\ 0.001641 & 0.001657 & 0.001641 & 0.001626 \\ 0.427011 & 0.427042 & 0.427042 & 0.427026 \end{pmatrix}$$

LOS unit vector X component in the OBACCF
LOS unit vector Y component in the OBACCF
LOS unit vector Z component in the OBACCF

Each column of the matrix Vlos_obaccf represents the x, y, and z components of a vector parallel to one of the measured LOSs and relative to the OBAC. The LOS vector is defined to point away from the HIRDLS instrument.

$$\Delta Alos\_obaccf_{i,j} := \left(acos\left(Vlos\_obaccf_{i,j}\right)\right) \cdot \frac{180}{\pi}$$

$$\Delta Alos\_obaccf = \begin{pmatrix} 154.721800 & 154.719796 & 154.719800 & 154.720804 \\ 89.905958 & 89.905056 & 89.905960 & 89.906863 \\ 64.722000 & 64.720000 & 64.720000 & 64.721000 \end{pmatrix}$$

Rotation of the IRCF relative to the OBACCF:

The matrix Alm_obac_dms contains results of HIRDLS OBAC alignment measurements performed by LM. The measured parameters are in degrees-minutes-seconds. The measured parameters are as labeled. Each parameter set was measured four times.

$$\text{Alm\_obac\_dms} := \begin{array}{c} \begin{array}{cccccccccccc} | & \text{Mar 02} & | & \text{May 02} & | & \text{Jul 02} & | & \text{Nov 02} & | \end{array} \\ \begin{pmatrix} 89 & 48 & 34 & 89 & 47 & 28 & 89 & 49 & 5 & 89 & 49 & 20 \\ 90 & 7 & 40 & 90 & 3 & 4 & 90 & 2 & 6 & 90 & 0 & 40 \\ 179 & 56 & 33 & 179 & 56 & 44 & 179 & 56 & 32 & 179 & 56 & 58 \end{pmatrix} \\ \begin{array}{cccccccccccc} | d & m & s & | d & m & s & | d & m & s & | d & m & s & | \end{array} \end{array}$$

Tilt of the OBAC secondary surface

Tilt of the OBAC primary surface

Horizontal angle between OBAC pri surf normal and IRCF -x axis

Tilt values less than 90 degrees indicate that the surface is tilted DOWNward with respect to vertical earth plumb. Tilt values greater than 90 degrees indicate that the surface is tilted UPward with respect to vertical earth plumb.

Converting the values in matrix Alm_obac_dms to decimal degrees gives:

$$k_j := -2 + j \cdot 3 \qquad \text{Alm\_obac}_{i,j} := \text{Alm\_obac\_dms}_{i,k_j} + \frac{\text{Alm\_obac\_dms}_{i,k_j+1}}{60} + \frac{\text{Alm\_obac\_dms}_{i,k_j+2}}{3600}$$

$$\text{Alm\_obac} = \begin{pmatrix} 89.809444 & 89.791111 & 89.818056 & 89.822222 \\ 90.127778 & 90.051111 & 90.035000 & 90.011111 \\ 179.942500 & 179.945556 & 179.942222 & 179.949444 \end{pmatrix}$$

Tilt of the OBAC secondary surface

Tilt of the OBAC primary surface

Horizontal angle between OBAC pri surf normal and IRCF -x axis

Converting the values in matrix Alm_obac to rotations about the OBACCF axes gives:

$$\text{ROTxyz}_{1,j} := \left(\text{Alm\_obac}_{1,j} - 90\right) \qquad \text{ROTxyz}_{2,j} := \left(\text{Alm\_obac}_{2,j} - 90\right) \qquad \text{ROTxyz}_{3,j} := \left(\text{Alm\_obac}_{3,j} - 180\right)$$

$$\text{ROTxyz} = \begin{pmatrix} -0.190556 & -0.208889 & -0.181944 & -0.177778 \\ 0.127778 & 0.051111 & 0.035000 & 0.011111 \\ -0.057500 & -0.054444 & -0.057778 & -0.050556 \end{pmatrix}$$

Rotation of the IRCF around the OBACCF x axis

Rotation of the IRCF around the OBACCF y axis

Rotation of the IRCF around the OBACCF z axis

Where positive values indicate a counter clockwise rotation around a given axis when looking down the positive side of the axis toward the origin.

$\boxed{\text{WRITEPRN}(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\ROTxyz"}) := \text{ROTxyz}}$

Coordinate transformation of the LOS from the OBACCF to the IRCF:

Next it will be necessary to convert unit vectors in the direction of the LOS in the OBACCF to unit vectors in the IRCF.

$\alpha_j := ROTxyz_{1,j} \cdot \frac{\pi}{180}$ $\beta_j := ROTxyz_{2,j} \cdot \frac{\pi}{180}$ $\gamma_j := ROTxyz_{3,j} \cdot \frac{\pi}{180}$

$$Tx_j := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_j) & \sin(\alpha_j) \\ 0 & -\sin(\alpha_j) & \cos(\alpha_j) \end{pmatrix}$$ Rotation about x axis $$Tx_1 = \begin{pmatrix} 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.999994 & -0.003326 \\ 0.000000 & 0.003326 & 0.999994 \end{pmatrix}$$

$$Ty_j := \begin{pmatrix} \cos(\beta_j) & 0 & -\sin(\beta_j) \\ 0 & 1 & 0 \\ \sin(\beta_j) & 0 & \cos(\beta_j) \end{pmatrix}$$ Rotation about y axis $$Ty_1 = \begin{pmatrix} 0.999998 & 0.000000 & -0.002230 \\ 0.000000 & 1.000000 & 0.000000 \\ 0.002230 & 0.000000 & 0.999998 \end{pmatrix}$$

$$Tz_j := \begin{pmatrix} \cos(\gamma_j) & \sin(\gamma_j) & 0 \\ -\sin(\gamma_j) & \cos(\gamma_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Rotation about z axis $$Tz_1 = \begin{pmatrix} 0.999999 & -0.001004 & 0.000000 \\ 0.001004 & 0.999999 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 \end{pmatrix}$$

$Txyz_j := Tx_j \cdot Ty_j \cdot Tz_j$ $$Txyz_1 = \begin{pmatrix} 0.999997 & -0.001004 & -0.002230 \\ 0.000996 & 0.999994 & -0.003326 \\ 0.002233 & 0.003324 & 0.999992 \end{pmatrix}$$

From a previous section:

$$Vlos\_obaccf = \begin{pmatrix} -0.904245 & -0.90423 & -0.90423 & -0.904238 \\ 0.001641 & 0.001657 & 0.001641 & 0.001626 \\ 0.427011 & 0.427042 & 0.427042 & 0.427026 \end{pmatrix}$$

Then: $Vlos\_ircf_j := Txyz_j \cdot Vlos\_obaccf^{\langle j \rangle}$ $Vlos\_ircf := augment(Vlos\_ircf_1, Vlos\_ircf_2, Vlos\_ircf_3, Vlos\_ircf_4)$

$$Vlos\_ircf = \begin{pmatrix} -0.905196 & -0.904612 & -0.904492 & -0.904322 \\ -0.000680 & -0.000756 & -0.000625 & -0.000497 \\ 0.424993 & 0.426236 & 0.426490 & 0.426852 \end{pmatrix}$$

Each column of the matrix Vlos_trcf contains the x, y, and z components of a unit vector parallel to one or the four measured LOSs and relative to the TRCF.

$$\Delta Alos\_ircf_{i,j} := \left(acos\left(Vlos\_ircf_{i,j}\right)\right) \cdot \frac{180}{\pi}$$

$$\Delta Alos\_ircf = \begin{pmatrix} 154.849738 & 154.771064 & 154.754960 & 154.732060 \\ 90.038937 & 90.043322 & 90.035802 & 90.028462 \\ 64.849772 & 64.771106 & 64.754989 & 64.732079 \end{pmatrix}$$

$\boxed{WRITEPRN(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Vlos_ircf"}) := Vlos\_ircf}$

Rotation of the IACCF relative to the IRCF:

The matrix Alm_iac_dms contains the results of HIRDLS IAC alignment measurements performed by LM. The measured parameters are in degrees-minutes-seconds. The measured parameters are as labeled. Each parameter set was measured four times.

$$\begin{array}{c} \begin{array}{cccccccccccc} | & \text{Mar 02} & | & \text{May 02} & | & \text{Jul 02} & | & \text{Nov 02} & | \end{array} \\ \text{Alm\_iac\_dms} := \begin{pmatrix} 89 & 59 & 13 & 89 & 56 & 46 & 89 & 55 & 38 & 89 & 55 & 20 \\ 90 & 12 & 9 & 90 & 14 & 56 & 90 & 13 & 43 & 90 & 15 & 45 \\ 149 & 56 & 26 & 149 & 55 & 51 & 149 & 55 & 57 & 149 & 56 & 11 \end{pmatrix} \\ \begin{array}{cccccccccccc} | d & m & s | & d & m & s | & d & m & s | & d & m & s | \end{array} \end{array}$$

Tilt of the IAC secondary surface

Tilt of the IAC primary surface

Horizontal angle between IAC pri surf and OBAC pri surface

Tilt values less than 90 degrees indicate that the surface is tilted DOWNward with respect to vertical earth plumb. Tilt values greater than 90 degrees indicate that the surface is tilted UPward with respect to vertical earth plumb.

Converting the values in matrix Alm_iac_dms to decimal degrees gives:

$$k_j := -2 + j \cdot 3 \qquad \text{Alm\_iac}_{i,j} := \text{Alm\_iac\_dms}_{i,k_j} + \frac{\text{Alm\_iac\_dms}_{i,k_j+1}}{60} + \frac{\text{Alm\_iac\_dms}_{i,k_j+2}}{3600}$$

$$\text{Alm\_iac} = \begin{pmatrix} 89.986944 & 89.946111 & 89.927222 & 89.922222 \\ 90.202500 & 90.248889 & 90.228611 & 90.262500 \\ 149.940556 & 149.930833 & 149.932500 & 149.936389 \end{pmatrix}$$

Tilt of the IAC secondary surface

Tilt of the IAC primary surface

Horizontal angle between IAC primary surf and OBAC primary surface

Converting the values in matrix Alm_iac to rotations about the TRCF axes gives:

$$\text{rotXYZ}_{1,j} := \left(\text{Alm\_iac}_{1,j} - 90\right) \qquad \text{rotXYZ}_{2,j} := \left(\text{Alm\_iac}_{2,j} - 90\right) \qquad \text{rotXYZ}_{3,j} := \text{Alm\_iac}_{3,j} - 180$$

$$\text{rotXYZ} = \begin{pmatrix} -0.013056 & -0.053889 & -0.072778 & -0.077778 \\ 0.202500 & 0.248889 & 0.228611 & 0.262500 \\ -30.059444 & -30.069167 & -30.067500 & -30.063611 \end{pmatrix}$$

Rotation of the IACCF around the TRCF x axis

Rotation of the IACCF around the TRCF y axis

Rotation of the IACCF around the TRCF z axis

Where positive values indicate a counter clockwise rotation around a given axis when looking down the positive side of the axis toward the origin.

Coordinate transformation of the LOS from the TRCF to IACCF:

Next it will be necessary to convert unit vectors in the direction of the LOS in the TRCF to unit vectors in the IACCF.

$\alpha_j := rotXYZ_{1,j} \cdot \frac{\pi}{180}$ $\beta_j := rotXYZ_{2,j} \cdot \frac{\pi}{180}$ $\gamma_j := rotXYZ_{3,j} \cdot \frac{\pi}{180}$

$$Tx_j := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_j) & \sin(\alpha_j) \\ 0 & -\sin(\alpha_j) & \cos(\alpha_j) \end{pmatrix}$$ Rotation about x axis $$Tx_1 = \begin{pmatrix} 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 1.000000 & -0.000228 \\ 0.000000 & 0.000228 & 1.000000 \end{pmatrix}$$

$$Ty_j := \begin{pmatrix} \cos(\beta_j) & 0 & -\sin(\beta_j) \\ 0 & 1 & 0 \\ \sin(\beta_j) & 0 & \cos(\beta_j) \end{pmatrix}$$ Rotation about y axis $$Ty_1 = \begin{pmatrix} 0.999994 & 0.000000 & -0.003534 \\ 0.000000 & 1.000000 & 0.000000 \\ 0.003534 & 0.000000 & 0.999994 \end{pmatrix}$$

$$Tz_j := \begin{pmatrix} \cos(\gamma_j) & \sin(\gamma_j) & 0 \\ -\sin(\gamma_j) & \cos(\gamma_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Rotation about z axis $$Tz_1 = \begin{pmatrix} 0.865506 & -0.500898 & 0.000000 \\ 0.500898 & 0.865506 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 \end{pmatrix}$$

$Txyz_j := Tx_j \cdot Ty_j \cdot Tz_j$ $$Txyz_1 = \begin{pmatrix} 0.865501 & -0.500895 & -0.003534 \\ 0.500898 & 0.865507 & -0.000228 \\ 0.003173 & -0.001573 & 0.999994 \end{pmatrix}$$

From a previous section:

$$Vlos\_ircf = \begin{pmatrix} -0.905196 & -0.904612 & -0.904492 & -0.904322 \\ -0.000680 & -0.000756 & -0.000625 & -0.000497 \\ 0.424993 & 0.426236 & 0.426490 & 0.426852 \end{pmatrix}$$

Then: $Vlos\_iaccf_j := Txyz_j \cdot Vlos\_ircf^{\langle j \rangle}$ $Vlos\_iaccf := augment(Vlos\_iaccf_1, Vlos\_iaccf_2, Vlos\_iaccf_3, Vlos\_iaccf_4)$

$$Vlos\_iaccf = \begin{pmatrix} -0.784610 & -0.784336 & -0.784162 & -0.784361 \\ -0.454096 & -0.454303 & -0.454247 & -0.454034 \\ 0.422119 & 0.422405 & 0.422788 & 0.422647 \end{pmatrix}$$

Each column of the matrix Vlos_iaccf contains the x, y, and z components of a unit vector parallel to one or the four measured LOSs and relative to the IACCF.

$$\Delta Alos\_iaccf_{i,j} := \left(acos\left(Vlos\_iaccf_{i,j}\right)\right) \cdot \frac{180}{\pi}$$

$$\Delta Alos\_iaccf = \begin{pmatrix} 141.684601 & 141.659272 & 141.643256 & 141.661662 \\ 117.006760 & 117.020110 & 117.016477 & 117.002801 \\ 65.031542 & 65.013460 & 64.989276 & 64.998210 \end{pmatrix}$$

Summarizing, the x, y, and z components of the LOS unit vectors in the IACCF are:

$$Vlos\_iaccf = \begin{pmatrix} -0.784610 & -0.784336 & -0.784162 & -0.784361 \\ -0.454096 & -0.454303 & -0.454247 & -0.454034 \\ 0.422119 & 0.422405 & 0.422788 & 0.422647 \end{pmatrix}$$

LOS unit vector X component in the IACCF

LOS unit vector Y component in the IACCF

LOS unit vector Z component in the IACCF

WRITEPRN("C:\Documents and Settings\tpedersen.EOS\My Documents\Vlos_iaccf") := Vlos_iaccf

Technical Memorandum

From: Tracy R. Pedersen, Swales Aerospace

To: Glenn B. Jackson, Claire L. Wilda

Date: April 27, 2004

Subject: **Verification of HIRDLS UIID Alignment Specification**

Document No: HIRDLS-TP-149C

Requirement:

HIRDLS alignment requirements, taken from the HIRDLS Unique Instrument Interface Document (UIID, Document # GSFC 424-28-21-06. Rev. B), are shown in the matrix Req. The given values represent the required knowledge of the maximum Line of Sight (LOS) to Instrument Alignment Cube (IAC) misalignment, in arcsec of rotation around the indicated axes. Since these rotations are 3σ zero to peak, the allowed peak to peak 3σ ranges of misalignment are double the values in matrix Req, and are shown in seconds in matrix Req2 and decimal degrees in matrix Req2dec.

$$Req := \begin{pmatrix} 180 \\ 180 \\ 360 \end{pmatrix} \quad \begin{matrix} \text{X axis (UIID 3.1.4.3)} \\ \text{Y axis (UIID 3.1.4.3)} \\ \text{Z axis (UIID 3.1.4.3)} \end{matrix} \qquad Req2 := 2 \cdot Req \qquad Req2 = \begin{pmatrix} 360 \\ 360 \\ 720 \end{pmatrix} \qquad Req2dec := \frac{Req2}{3600} \qquad Req2dec = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.2 \end{pmatrix}$$

LOS data:

Vlos_iaccf is a matrix containing reduced HIRDLS Line of Sight (LOS) to Instrument Alignment Cube (IAC) alignment vectors, in decimal degrees, taken from Technical Memorandum HIRDLS-TP-148C.

$$\boxed{Vlos\_iaccf := READPRN(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Vlos\_iaccf"})}$$

$$Vlos\_iaccf = \begin{pmatrix} -0.784610 & -0.784336 & -0.784162 & -0.784361 \\ -0.454096 & -0.454303 & -0.454247 & -0.454034 \\ 0.422119 & 0.422405 & 0.422788 & 0.422647 \end{pmatrix} \quad \begin{matrix} \text{LOS unit vector X component in the IACCF} \\ \text{LOS unit vector Y component in the IACCF} \\ \text{LOS unit vector Z component in the IACCF} \end{matrix}$$

n is the number of samples: $n := 4 \qquad j := 1, 2 .. n$

m is the number of measured parameters: $m := 3 \qquad i := 1, 2 .. m$

The vectors in matrix Vlos_iaccf are converted to rotations around the axes of the IACCF as follows:

$$rotX_j := atan\left(\frac{Vlos\_iaccf_{3,j}}{Vlos\_iaccf_{2,j}}\right) \qquad rotY_j := atan\left(\frac{Vlos\_iaccf_{3,j}}{Vlos\_iaccf_{1,j}}\right) \qquad rotZ_j := atan\left(\frac{Vlos\_iaccf_{2,j}}{Vlos\_iaccf_{1,j}}\right)$$

$$rotXYZ := stack\left(rotX^T, rotY^T, rotZ^T\right) \cdot \frac{180}{\pi}$$

$$rotXYZ = \begin{pmatrix} -42.910 & -42.916 & -42.946 & -42.950 \\ -28.280 & -28.305 & -28.332 & -28.318 \\ 30.060 & 30.080 & 30.083 & 30.065 \end{pmatrix} \quad \begin{matrix} \text{Rotation of the LOS unit vector around the IACCF X axis} \\ \text{Rotation of the LOS unit vector around the IACCF Y axis} \\ \text{Rotation of the LOS unit vector around the IACCF Z axis} \end{matrix}$$

Statistical analysis:

The number of samples of each alignment measurement, four, is quite small. For such cases, even though the underlying population may be Gaussian, a Gaussian distribution cannot be used to accurately derive probability intervals. In such cases, a t-distribution is often used to model the sample population. A t-distribution will be used here.

The mean value, m, of each parameter is:

$$m_i := \frac{1}{n} \cdot \sum_{j=1}^{n} rotXYZ_{i,j} \qquad m = \begin{pmatrix} -42.930 \\ -28.309 \\ 30.072 \end{pmatrix}$$

The standard deviation, s, of each parameter is:

$$s_i := \left[\frac{1}{n-1} \cdot \sum_{j=1}^{n} \left(rotXYZ_{i,j} - m_i \right)^2 \right]^{0.5} \qquad s = \begin{pmatrix} 0.02 \\ 0.022 \\ 0.011 \end{pmatrix}$$

The degree of freedom, f, is:

$$f := n - 1 \qquad f = 3.000$$

The mean occurrence rate, v, is:

$$v(i) := \frac{n}{m_i} \qquad v(i) = \begin{pmatrix} -0.093 \\ -0.141 \\ 0.133 \end{pmatrix}$$

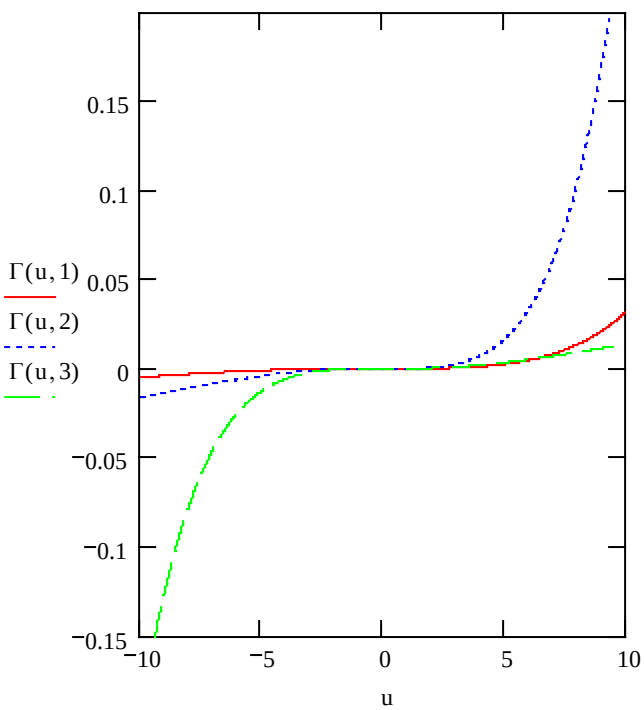
The Gamma probability density function, Γ, is:

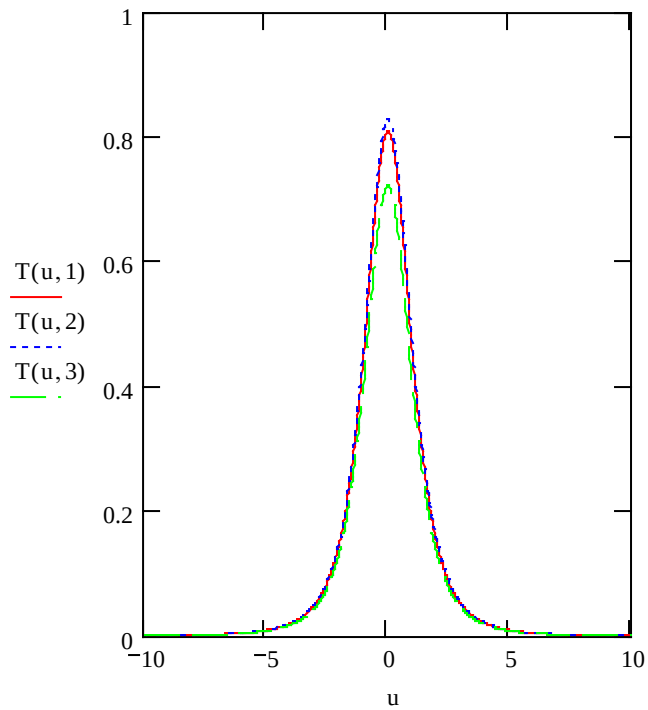
$$\Gamma(u,i) := \frac{v(i) \cdot (v(i) \cdot u)^{n-1}}{(n-1)!} \cdot e^{-v(i) \cdot u}$$

Where u is a variable representing interval.

The T probability density function, T, is:

$$T(u,i) := \frac{\Gamma\left(\frac{f+1}{2}, i\right)}{(\pi \cdot f)^{0.5} \cdot \Gamma\left(\frac{f}{2}, i\right)} \cdot \left(1 + \frac{u^2}{f}\right)^{\frac{-1}{2} \cdot (f+1)}$$





The probability, P, that the value of a parameter will be in the interval -u to u is:

$$P(u,i) := \frac{\int_{-u}^{u} T(u,i)\, du}{\int_{-100}^{100} T(u,i)\, du}$$

To solve for the probability that the instrument alignment will be within the specified interval:

Solving for u using standard formulas for interval estimation gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 0.8 \\ \text{while } 2 \cdot u_i \cdot \dfrac{s_i}{n^{0.5}} \le Req2dec_i \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u = \begin{pmatrix} 4.961 \\ 4.566 \\ 17.929 \end{pmatrix}$$

Solving for P gives:

$$P(u_i, i) = \begin{pmatrix} 0.984 \\ 0.98 \\ 1 \end{pmatrix}$$

Probability that X axis rotation will fall within the specified interval of
Probability that Y axis rotation will fall within the specified interval of
Probability that Z axis rotation will fall within the specified interval of

$$Req2 = \begin{pmatrix} 360 \\ 360 \\ 720 \end{pmatrix} \text{ arcsec}$$

$Prob_i := P(u_i, i)$ $\quad$ WRITEPRN("C:\Documents and Settings\tpedersen.EOS\My Documents\Prob") := Prob

Again using standard formulas for interval estimation:

$$Imin_i := m_i - u_i \cdot \frac{s_i}{n^{0.5}} \qquad Imax_i := m_i + u_i \cdot \frac{s_i}{n^{0.5}} \qquad Iavg_i := \frac{Imin_i + Imax_i}{2} \qquad Idif_i := Imax_i - Imin_i$$

$$Imin = \begin{pmatrix} -42.980 \\ -28.359 \\ 29.972 \end{pmatrix} \quad Imax = \begin{pmatrix} -42.880 \\ -28.259 \\ 30.172 \end{pmatrix} \quad Iavg = \begin{pmatrix} -42.930 \\ -28.309 \\ 30.072 \end{pmatrix} \quad Idif = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.2 \end{pmatrix}$$

Converting these results from decimal degrees to degrees-minutes-seconds gives:

$$aImin_i := if(Imin_i < 0, -Imin_i, Imin_i) \qquad aImax_i := if(Imax_i < 0, -Imax_i, Imax_i) \qquad aIavg_i := if(Iavg_i < 0, -Iavg_i, Iavg_i)$$

$$dmin_i := floor(aImin_i) \qquad mmin_i := floor\left[(aImin_i - dmin_i) \cdot 60\right] \qquad smin_i := \left(aImin_i - dmin_i - \frac{mmin_i}{60}\right) \cdot 3600$$

$$dmax_i := floor(aImax_i) \qquad mmax_i := floor\left[(aImax_i - dmax_i) \cdot 60\right] \qquad smax_i := \left(aImax_i - dmax_i - \frac{mmax_i}{60}\right) \cdot 3600$$

$$davg_i := floor(aIavg_i) \qquad mavg_i := floor\left[(aIavg_i - davg_i) \cdot 60\right] \qquad savg_i := \left(aIavg_i - davg_i - \frac{mavg_i}{60}\right) \cdot 3600$$

$$Imindms := augment(dmin, mmin, smin) \qquad Imaxdms := augment(dmax, mmax, smax)$$

$$Iavgdms := augment(davg, mavg, savg) \qquad Idifdms := Idif \cdot 3600 \qquad o := 3 \qquad k := 1, 2 .. o$$

$$Imindms_{i,k} := if(Imin_i < 0, -Imindms_{i,k}, Imindms_{i,k}) \qquad Imaxdms_{i,k} := if(Imax_i < 0, -Imaxdms_{i,k}, Imaxdms_{i,k})$$

$$Iavgdms_{i,k} := if(Iavg_i < 0, -Iavgdms_{i,k}, Iavgdms_{i,k})$$

$$Imindms = \begin{pmatrix} -42 & -58 & -49 \\ -28 & -21 & -31 \\ 29 & 58 & 19 \end{pmatrix} \quad Imaxdms = \begin{pmatrix} -42 & -52 & -49 \\ -28 & -15 & -31 \\ 30 & 10 & 19 \end{pmatrix} \quad Iavgdms = \begin{pmatrix} -42 & -55 & -49 \\ -28 & -18 & -31 \\ 30 & 4 & 19 \end{pmatrix} \quad Idifdms = \begin{pmatrix} 360 \\ 360 \\ 720 \end{pmatrix} \text{ arcsec}$$

Note that matrix Idifdms is equal to matrix Req2, which contains the LOS to IRCF alignment specifications. Thus, the accuracy of the above calculations is demonstrated.

To solve for the interval required to ensure that a 0.9986 confidence level is achieved: $CL := 0.9986$

Solving for u using the probability function above gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 11.52 \\ \text{while } P(u_i, i) \le CL \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u = \begin{pmatrix} 11.526 \\ 11.526 \\ 11.526 \end{pmatrix}$$

Using standard formulas for interval estimation:

$$Imin_i := m_i - u_i \cdot \frac{s_i}{n^{0.5}} \qquad Imax_i := m_i + u_i \cdot \frac{s_i}{n^{0.5}} \qquad Iavg_i := \frac{Imin_i + Imax_i}{2} \qquad Idif_i := Imax_i - Imin_i$$

$$Imin = \begin{pmatrix} -43.047 \\ -28.435 \\ 30.008 \end{pmatrix} \qquad Imax = \begin{pmatrix} -42.814 \\ -28.182 \\ 30.136 \end{pmatrix} \qquad Iavg = \begin{pmatrix} -42.930 \\ -28.309 \\ 30.072 \end{pmatrix} \qquad Idif = \begin{pmatrix} 0.232 \\ 0.252 \\ 0.129 \end{pmatrix}$$

Converting these results from decimal degrees to degrees-minutes-seconds gives:

$$aImin_i := if(Imin_i < 0, -Imin_i, Imin_i) \qquad aImax_i := if(Imax_i < 0, -Imax_i, Imax_i) \qquad aIavg_i := if(Iavg_i < 0, -Iavg_i, Iavg_i)$$

$$dmin_i := floor(aImin_i) \qquad mmin_i := floor\left[(aImin_i - dmin_i) \cdot 60\right] \qquad smin_i := \left(aImin_i - dmin_i - \frac{mmin_i}{60}\right) \cdot 3600$$

$$dmax_i := floor(aImax_i) \qquad mmax_i := floor\left[(aImax_i - dmax_i) \cdot 60\right] \qquad smax_i := \left(aImax_i - dmax_i - \frac{mmax_i}{60}\right) \cdot 3600$$

$$davg_i := floor(aIavg_i) \qquad mavg_i := floor\left[(aIavg_i - davg_i) \cdot 60\right] \qquad savg_i := \left(aIavg_i - davg_i - \frac{mavg_i}{60}\right) \cdot 3600$$

$$Imindms := augment(dmin, mmin, smin) \qquad Imaxdms := augment(dmax, mmax, smax)$$

$$Iavgdms := augment(davg, mavg, savg) \qquad Idifdms9986 := Idif \cdot 3600 \qquad o := 3 \qquad k := 1, 2 .. o$$

$$Imindms_{i,k} := if(Imin_i < 0, -Imindms_{i,k}, Imindms_{i,k}) \qquad Imaxdms_{i,k} := if(Imax_i < 0, -Imaxdms_{i,k}, Imaxdms_{i,k})$$

$$Iavgdms_{i,k} := if(Iavg_i < 0, -Iavgdms_{i,k}, Iavgdms_{i,k})$$

$$Imindms = \begin{pmatrix} -43 & -2 & -48 \\ -28 & -26 & -5 \\ 30 & 0 & 28 \end{pmatrix} \qquad Imaxdms = \begin{pmatrix} -42 & -48 & -51 \\ -28 & -10 & -56 \\ 30 & 8 & 11 \end{pmatrix} \qquad Iavgdms = \begin{pmatrix} -42 & -55 & -49 \\ -28 & -18 & -31 \\ 30 & 4 & 19 \end{pmatrix} \qquad Idifdms9986 = \begin{pmatrix} 836 \\ 909 \\ 463 \end{pmatrix} \text{ arcsec}$$

WRITEPRN("C:\Documents and Settings\tpedersen.EOS\My Documents\Iavgdms") := Iavgdms

WRITEPRN("C:\Documents and Settings\tpedersen.EOS\My Documents\Idifdms9986") := Idifdms9986

To solve for the interval required to ensure that a 0.99 confidence level is achieved: $CL := 0.99$

Solving for u using the probability function above gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 5.84 \\ \text{while } P(u_i, i) \le CL \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u = \begin{pmatrix} 5.841 \\ 5.841 \\ 5.841 \end{pmatrix}$$

Using standard formulas for interval estimation:

$$Imin_i := m_i - u_i \cdot \frac{s_i}{n^{0.5}} \qquad Imax_i := m_i + u_i \cdot \frac{s_i}{n^{0.5}} \qquad Iavg_i := \frac{Imin_i + Imax_i}{2} \qquad Idif_i := Imax_i - Imin_i$$

$$Imin = \begin{pmatrix} -42.989 \\ -28.373 \\ 30.039 \end{pmatrix} \qquad Imax = \begin{pmatrix} -42.872 \\ -28.245 \\ 30.105 \end{pmatrix} \qquad Iavg = \begin{pmatrix} -42.930 \\ -28.309 \\ 30.072 \end{pmatrix} \qquad Idif = \begin{pmatrix} 0.118 \\ 0.128 \\ 0.065 \end{pmatrix}$$

Converting these results from decimal degrees to degrees-minutes-seconds gives:

$$aImin_i := if(Imin_i < 0, -Imin_i, Imin_i) \qquad aImax_i := if(Imax_i < 0, -Imax_i, Imax_i) \qquad aIavg_i := if(Iavg_i < 0, -Iavg_i, Iavg_i)$$

$$dmin_i := floor(aImin_i) \qquad mmin_i := floor\left[(aImin_i - dmin_i) \cdot 60\right] \qquad smin_i := \left(aImin_i - dmin_i - \frac{mmin_i}{60}\right) \cdot 3600$$

$$dmax_i := floor(aImax_i) \qquad mmax_i := floor\left[(aImax_i - dmax_i) \cdot 60\right] \qquad smax_i := \left(aImax_i - dmax_i - \frac{mmax_i}{60}\right) \cdot 3600$$

$$davg_i := floor(aIavg_i) \qquad mavg_i := floor\left[(aIavg_i - davg_i) \cdot 60\right] \qquad savg_i := \left(aIavg_i - davg_i - \frac{mavg_i}{60}\right) \cdot 3600$$

$$Imindms := augment(dmin, mmin, smin) \qquad Imaxdms := augment(dmax, mmax, smax)$$

$$Iavgdms := augment(davg, mavg, savg) \qquad Idifdms := Idif \cdot 3600 \qquad o := 3 \qquad k := 1, 2 .. o$$

$$Imindms_{i,k} := if(Imin_i < 0, -Imindms_{i,k}, Imindms_{i,k}) \qquad Imaxdms_{i,k} := if(Imax_i < 0, -Imaxdms_{i,k}, Imaxdms_{i,k})$$

$$Iavgdms_{i,k} := if(Iavg_i < 0, -Iavgdms_{i,k}, Iavgdms_{i,k})$$

$$Imindms = \begin{pmatrix} -42 & -59 & -21 \\ -28 & -22 & -21 \\ 30 & 2 & 22 \end{pmatrix} \quad Imaxdms = \begin{pmatrix} -42 & -52 & -17 \\ -28 & -14 & -41 \\ 30 & 6 & 17 \end{pmatrix} \quad Iavgdms = \begin{pmatrix} -42 & -55 & -49 \\ -28 & -18 & -31 \\ 30 & 4 & 19 \end{pmatrix} \quad Idifdms = \begin{pmatrix} 424 \\ 461 \\ 235 \end{pmatrix} \text{ arcsec}$$

To solve for the interval required to ensure that a 0.98 confidence level is achieved: $CL := 0.98$

Solving for u using the probability function above gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 4.54 \\ \text{while } P(u_i, i) \le CL \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u = \begin{pmatrix} 4.541 \\ 4.541 \\ 4.541 \end{pmatrix}$$

Using standard formulas for interval estimation:

$$Imin_i := m_i - u_i \cdot \frac{s_i}{n^{0.5}} \qquad Imax_i := m_i + u_i \cdot \frac{s_i}{n^{0.5}} \qquad Iavg_i := \frac{Imin_i + Imax_i}{2} \qquad Idif_i := Imax_i - Imin_i$$

$$Imin = \begin{pmatrix} -42.976 \\ -28.358 \\ 30.047 \end{pmatrix} \qquad Imax = \begin{pmatrix} -42.885 \\ -28.259 \\ 30.097 \end{pmatrix} \qquad Iavg = \begin{pmatrix} -42.930 \\ -28.309 \\ 30.072 \end{pmatrix} \qquad Idif = \begin{pmatrix} 0.092 \\ 0.099 \\ 0.051 \end{pmatrix}$$

Converting these results from decimal degrees to degrees-minutes-seconds gives:

$$aImin_i := if(Imin_i < 0, -Imin_i, Imin_i) \qquad aImax_i := if(Imax_i < 0, -Imax_i, Imax_i) \qquad aIavg_i := if(Iavg_i < 0, -Iavg_i, Iavg_i)$$

$$dmin_i := floor(aImin_i) \qquad mmin_i := floor\left[(aImin_i - dmin_i) \cdot 60\right] \qquad smin_i := \left(aImin_i - dmin_i - \frac{mmin_i}{60}\right) \cdot 3600$$

$$dmax_i := floor(aImax_i) \qquad mmax_i := floor\left[(aImax_i - dmax_i) \cdot 60\right] \qquad smax_i := \left(aImax_i - dmax_i - \frac{mmax_i}{60}\right) \cdot 3600$$

$$davg_i := floor(aIavg_i) \qquad mavg_i := floor\left[(aIavg_i - davg_i) \cdot 60\right] \qquad savg_i := \left(aIavg_i - davg_i - \frac{mavg_i}{60}\right) \cdot 3600$$

$$Imindms := augment(dmin, mmin, smin) \qquad Imaxdms := augment(dmax, mmax, smax)$$

$$Iavgdms := augment(davg, mavg, savg) \qquad Idifdms := Idif \cdot 3600 \qquad o := 3 \qquad k := 1, 2 .. o$$

$$Imindms_{i,k} := if(Imin_i < 0, -Imindms_{i,k}, Imindms_{i,k}) \qquad Imaxdms_{i,k} := if(Imax_i < 0, -Imaxdms_{i,k}, Imaxdms_{i,k})$$

$$Iavgdms_{i,k} := if(Iavg_i < 0, -Iavgdms_{i,k}, Iavgdms_{i,k})$$

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$$Imindms = \begin{pmatrix} -42 & -58 & -34 \\ -28 & -21 & -30 \\ 30 & 2 & 48 \end{pmatrix} \quad Imaxdms = \begin{pmatrix} -42 & -53 & -5 \\ -28 & -15 & -32 \\ 30 & 5 & 50 \end{pmatrix} \quad Iavgdms = \begin{pmatrix} -42 & -55 & -49 \\ -28 & -18 & -31 \\ 30 & 4 & 19 \end{pmatrix} \quad Idifdms = \begin{pmatrix} 330 \\ 358 \\ 182 \end{pmatrix} \quad \text{arcsec}$$

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To solve for the probability that the alignment will fall within the three-sigma, peak-to-peak, interval:

From above, the values of the standard deviation, sigma, are:

$$s = \begin{pmatrix} 0.020 \\ 0.022 \\ 0.011 \end{pmatrix} \text{ degrees} \qquad s\_dms := s \cdot 3600 \qquad s\_dms = \begin{pmatrix} 73 \\ 79 \\ 40 \end{pmatrix} \text{ arcsec}$$

The six-sigma peak-to-peak intervals are:

$$I3\sigma\_pp := 6 \cdot s \qquad I3\sigma\_pp = \begin{pmatrix} 0.121 \\ 0.131 \\ 0.067 \end{pmatrix} \text{ degrees} \qquad I3\sigma\_pp\_dms := I3\sigma\_pp \cdot 3600 \qquad I3\sigma\_pp\_dms = \begin{pmatrix} 435 \\ 473 \\ 241 \end{pmatrix} \text{ arcsec}$$

Solving for u using standard formulas for interval estimation gives:

$$u_i := \left| \begin{array}{l} u_i \leftarrow 5.9 \\ \text{while } 2 \cdot u_i \cdot \dfrac{s_i}{n^{0.5}} \le I3\sigma\_pp_i \\ \quad u_i \leftarrow u_i + 0.001 \end{array} \right. \qquad u = \begin{pmatrix} 6.001 \\ 6.001 \\ 6.001 \end{pmatrix}$$

Solving for P gives:

$$P(u_i, i) = \begin{pmatrix} 0.991 \\ 0.991 \\ 0.991 \end{pmatrix}$$

Probability that X axis rotation will fall within the 3σ p-p interval of
Probability that Y axis rotation will fall within the 3σ p-p interval of
Probability that Z axis rotation will fall within the 3σ p-p interval of

$$I3\sigma\_pp\_dms = \begin{pmatrix} 435 \\ 473 \\ 241 \end{pmatrix} \text{arcsec}$$

Again using standard formulas for interval estimation:

$$Imin_i := m_i - u_i \cdot \frac{s_i}{n^{0.5}} \qquad Imax_i := m_i + u_i \cdot \frac{s_i}{n^{0.5}} \qquad Iavg_i := \frac{Imin_i + Imax_i}{2} \qquad Idif_i := Imax_i - Imin_i$$

$$Imin = \begin{pmatrix} -42.991 \\ -28.374 \\ 30.039 \end{pmatrix} \qquad Imax = \begin{pmatrix} -42.870 \\ -28.243 \\ 30.105 \end{pmatrix} \qquad Iavg = \begin{pmatrix} -42.930 \\ -28.309 \\ 30.072 \end{pmatrix} \qquad Idif = \begin{pmatrix} 0.121 \\ 0.131 \\ 0.067 \end{pmatrix}$$

Converting these results from decimal degrees to degrees-minutes-seconds gives:

$$aImin_i := if(Imin_i < 0, -Imin_i, Imin_i) \qquad aImax_i := if(Imax_i < 0, -Imax_i, Imax_i) \qquad aIavg_i := if(Iavg_i < 0, -Iavg_i, Iavg_i)$$

$$dmin_i := floor(aImin_i) \qquad mmin_i := floor\left[(aImin_i - dmin_i) \cdot 60\right] \qquad smin_i := \left(aImin_i - dmin_i - \frac{mmin_i}{60}\right) \cdot 3600$$

$$dmax_i := floor(aImax_i) \qquad mmax_i := floor\left[(aImax_i - dmax_i) \cdot 60\right] \qquad smax_i := \left(aImax_i - dmax_i - \frac{mmax_i}{60}\right) \cdot 3600$$

$$davg_i := floor(aIavg_i) \qquad mavg_i := floor\left[(aIavg_i - davg_i) \cdot 60\right] \qquad savg_i := \left(aIavg_i - davg_i - \frac{mavg_i}{60}\right) \cdot 3600$$

$$Imindms := augment(dmin, mmin, smin) \qquad Imaxdms := augment(dmax, mmax, smax)$$

$$Iavgdms := augment(davg, mavg, savg) \qquad Idifdms3spp := Idif \cdot 3600 \qquad o := 3 \qquad k := 1, 2 .. o$$

$$Imindms_{i,k} := if\left(Imin_i < 0, -Imindms_{i,k}, Imindms_{i,k}\right) \qquad Imaxdms_{i,k} := if\left(Imax_i < 0, -Imaxdms_{i,k}, Imaxdms_{i,k}\right)$$

$$Iavgdms_{i,k} := if\left(Iavg_i < 0, -Iavgdms_{i,k}, Iavgdms_{i,k}\right)$$

$$Imindms = \begin{pmatrix} -42 & -59 & -27 \\ -28 & -22 & -27 \\ 30 & 2 & 19 \end{pmatrix} \quad Imaxdms = \begin{pmatrix} -42 & -52 & -12 \\ -28 & -14 & -34 \\ 30 & 6 & 20 \end{pmatrix} \quad Iavgdms = \begin{pmatrix} -42 & -55 & -49 \\ -28 & -18 & -31 \\ 30 & 4 & 19 \end{pmatrix} \quad Idifdms3spp = \begin{pmatrix} 435 \\ 473 \\ 241 \end{pmatrix} \text{ arcsec}$$

WRITEPRN("C:\Documents and Settings\tpedersen.EOS\My Documents\Idifdms3szp") := Idifdms3spp

Technical Memorandum

From: Tracy R. Pedersen, Swales Aerospace

To: Glenn B. Jackson, Claire L. Wilda

Date: February 6, 2004

Subject: **Summary of HIRDLS Pointing Accuracy and Knowledge**

Document No: HIRDLS-TP-150A

Per requirements in the HIRDLS Interface Control Document (ICD, TRW document # D26477, Rev. D) and the HIRDLS Unique Instrument Interface Document (UIID, GSFC document # GSFC 424-2106), below is a summary of pointing accuracy and knowledge values for the HIRDLS instrument. The data below is taken from HIRDLS Technical Memoranda: HIRDLS-TP-148C and HIRDLS-TP-149C.

$l := 3$ $\quad i := 1, 2 .. l$ $\quad$ l is the number of parameters

$n := 4$ $\quad j := 1, 2 .. n$ $\quad$ n is the number of samples

The X, Y, and Z components of the average LOS unit vector in the IAC coordinate frame are

$\boxed{\text{Vlos\_iaccf} := \text{READPRN}(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Vlos\_iaccf"})}$

$$\text{Vlos\_iaccf} = \begin{pmatrix} -0.784610 & -0.784336 & -0.784162 & -0.784361 \\ -0.454096 & -0.454303 & -0.454247 & -0.454034 \\ 0.422119 & 0.422405 & 0.422788 & 0.422647 \end{pmatrix} \qquad \text{Vlos\_iaccf\_avg}_i := \frac{\sum_{j=1}^{4} \text{Vlos\_iaccf}_{i,j}}{4}$$

$$\text{Vlos\_iaccf\_avg} = \begin{pmatrix} -0.784367 \\ -0.454170 \\ 0.422490 \end{pmatrix}$$

Average LOS unit vector x axis component in IAC coord frame

Average LOS unit vector y axis component in IAC coord frame

Average LOS unit vector z axis component in IAC coord frame

The average rotation of the LOS around the IAC coordinate frame axes are:

$\boxed{\text{Iavgdms} := \text{READPRN}(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Iavgdms"})}$

$$\text{Iavgdms} = \begin{pmatrix} -42 & -55 & -49 \\ -28 & -18 & -31 \\ 30 & 4 & 19 \end{pmatrix}$$

Average rotation around IAC x axis

Average rotation around IAC y axis $\quad$ deg min sec

Average rotation around IAC z axis

The intervals of rotation of the LOS about the IAC coordinate frame axes which provide a 0.9986 confidence level are:

$\boxed{\text{Iintdms9986} := \text{READPRN}(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Idifdms9986"})}$

$$\text{Iintdms9986} = \begin{pmatrix} 836 \\ 909 \\ 463 \end{pmatrix}$$

Interval for 0.9986 confidence level on x axis rotation

Interval for 0.9986 confidence level on y axis rotation $\quad$ arcsec p-p

Interval for 0.9986 confidence level on z axis rotation

The intervals of rotation of the LOS about the IAC coordinate frame axes which provide a three-sigma, zero-to-peak, confidence level are:

$$\boxed{\text{Iintdms3}\sigma\text{zp} := \text{READPRN}(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Idifdms3szp"})}$$

$$\text{Iintdms3}\sigma\text{zp} = \begin{pmatrix} 436 \\ 473 \\ 241 \end{pmatrix}$$

Interval for three-sigma confidence level on x axis rotation

Interval for three-sigma confidence level on y axis rotation

Interval for three-sigma confidence level on z axis rotation

arcsec

The probability that the rotations of the LOS about the IAC coordinate frame axes will fall within the specified intervals are:

$$\boxed{\text{Prob} := \text{READPRN}(\text{"C:\Documents and Settings\tpedersen.EOS\My Documents\Prob"})}$$

$$\text{Prob} = \begin{pmatrix} 0.984 \\ 0.980 \\ 1.000 \end{pmatrix}$$

Probability that the X axix rotation will fall within the specified interval of 360 arcsec

Probability that the Y axix rotation will fall within the specified interval of 360 arcsec

Probability that the Z axix rotation will fall within the specified interval of 720 arcsec

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Notes: 1) The LOS unit vector points away from the instrument.

2) Positive rotation values indicate a counterclockwise rotation around a given axis when looking down the positive side of the axis toward the origin.